Stern Math: Theory and Practice

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Multisensory Mathematics Instruction

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This chapter is devoted to the description of the *Structural Arithmetic* (Stern & Stern, 1971) method of teaching mathematical concepts, in which multisensory materials are used. Thus, children achieve mastery of arithmetic without being forced to learn only by memorization and parroting. These materials are designed to enable children to learn on their own; after giving such materials to her students, one first-grade teacher cried, "They teach themselves!"

A structured, multisensory teaching approach is of special importance to children with learning disabilities. These children have difficulty with language concepts and associations and memory. They are usually struggling with a combination of these deficits and may also have difficulties with attention. To understand concepts, students with learning disabilities must learn to receive and integrate information from as many different senses as possible. They often have difficulty in "getting things" from books; however, they begin to trust their ability to learn when they have real experiences with multisensory materials. Because Structural Arithmetic is a multisensory approach, it enables children to develop concepts through experimenting with materials on their own.

The materials described in this chapter convey concepts through structures that are true to the mathematical relationships being taught. To understand these relationships, children pick up the materials and measure and compare them. A concept or a quantity is not presented in isolation but in context so that children can explore and discover the many different mathematical relationships that are possible. Good multisensory materials not only will stimulate such activities but also should present concepts so vividly and clearly that children can visualize them later. More than 100 sequenced activities and games have been gathered together in teacher's guides titled *Experimenting with Numbers: A Guide for Preschool, Kindergarten, and First Grade Teachers* (Stern, 1988) and *Structural Arithmetic Workbooks 1-3 and Teachers' Guides* (Stern & Gould, 1988-1992). They are the result of the further development of the original work published in Children Discover Arithmetic: *An Introduction to Structural Arithmetic* (Stern & Stern, 1971). These materials have been used successfully in classrooms for more than 45 years. Teachers also have expressed their appreciation for the illustrated directions that make teaching with multisensory structured materials much easier.

What is involved in the formation of concepts? Children seem to reason with mental pictures. Therefore, when teaching children to think, we must develop their ability to form images. Multisensory materials will have fulfilled their purpose when the children can visualize the concepts presented. It is not sufficient, however, for them to visualize a quantity in isolation; they must be able to visualize it in relation to other numbers and turn it around in their minds, and they must understand the actions that can be performed on it. For example, a teacher turned face down a pattern board with blanks for eight cubes and asked Sean to build the cube pattern he thinks will fit into it. When Sean built the 8-pattern correctly, he explained, "I imagined it in my brain, then I built it, then I turned the board over, and the cubes all fit!"

In addition to mental pictures, language plays a crucial role in the formation of concepts. The best way to teach children the meaning of spoken language is to give them the opportunity to see and touch what the words describe and, thus, work out for themselves what the words mean. In many of the math games in this chapter, children are asked to carry out spoken directions. This develops both their receptive language and their auditory memory. It is especially important for children with language deficits to develop these abilities.

In mathematics, as in any other subject, language is a vehicle for thought; therefore, students need many opportunities to put newly discovered concepts into their own words. Parroting the words of teachers is not a sign of true learning. Instead, the teacher should often encourage students to talk about concepts by asking them, "How did you figure out your answer?"

In the Structural Arithmetic approach, the children progress from activities and games with concrete materials that help them form basic concepts to the final steps of recording addition and subtraction facts with number symbols, or numerals. Here are the steps of Structural Arithmetic, stated as goals and objectives, to be followed in the first years of mathematics instruction:

- *Level I* To discover size relationships between amounts from 1 to 10, to know where each comes in the sequence from 1 to 10, to recognize odd and even numbers built as cube patterns, and to form number combinations with sums ranging from 1 to 10 using number blocks
- *Level II* To learn number names and how to count from 1 to 10, to understand terms such as *bigger than* and *smaller than*, and to recite the addition facts with sums of 10 or less that have been discovered with the materials
- *Level III* To identify and write the numerals 1-10 and 0, to understand the amount each numeral stands for, to understand the meaning of addition or subtraction equations, to write the answers to addition facts with sums of 10 or less and the answers to their related subtraction facts, and to demonstrate with materials the solution of a word problem and then record the addition or subtraction equation used to solve it

At Levels I and II the amounts from 1 to 10 are introduced in two different ways: with groups of *cubes* to be counted and with *number blocks*. For example, the amount of 4 is introduced both with 4 single cubes and by a number block called a *4-block*, which looks like four cubes glued together in a straight line. Number symbols or numerals are introduced later on. Children realize that they can name a small number block of 2 or 3 units at a glance but that they need to verify the name of a longer number block of 8 or 9 units by counting its units. They soon realize that each number block is easily recognized by its relative size, its position in the sequence from 1 to 10, and its color (a peripheral characteristic that is not mathematical). For example, children learn that the 10-block is the biggest number block, stands last in the sequence from 1 to 10, and is blue. The 9-block thus is the "next-to-biggest" number block, comes just before the last number block, and is black. These characteristics enable children to identify each number and study its changing role in different situations.

First Experiments: The Counting Board

Children begin their explorations of number concepts by fitting cubes or number blocks into grooves of the *counting board* (see Figure 11.1). Using the board, they carry out many experiments on their own. Concrete materials alone, however, do not lead to the development of mathematical thinking. The number blocks or cubes must be looked at, picked up, compared, and fit into activities that have been fashioned so as to make clear the structure of our base-10 number system. Teachers as well as children must put discoveries and relationships into words. Once the students understand these concepts, the teacher will introduce the symbols that stand for them (see Number Symbols and Signs: Level III).

Number Concepts and Language: Levels I and II

By experimenting with materials, children discover the many things they can do with numbers. Soon they are impatient to tell each other about their discoveries.

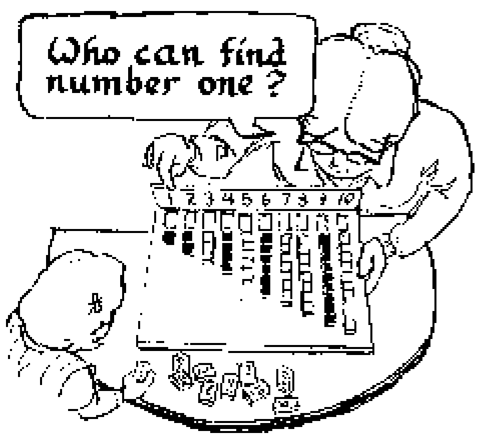

Figure 11.1. The counting board: matching number markers to numerals on the number guide.

Once they have learned the number names of the blocks, they begin to express their thoughts in language that comes to them naturally. (For example, Sumi might point to a 5-block and a 3-block and say, "I am 5, but my brother is only 3.") When children express ideas about numbers in their own words, they will have a more secure understanding of the language of mathematics when they hear it later. Teachers who listen to their students' words are often alerted to difficulties in comprehension and are able to give help before the problems cause even more trouble. In more formal environments, teachers give children orders to carry out. In doing so, children show whether they understand new vocabulary. For instance, a teacher might say, "Add

3 and 2, and tell me what 3 plus 2 equals." Students respond by adding together a 3-block and a 2-block and placing them alongside the 5-block. They see that 3 and 2 are "the same size as 5," which they learn is expressed by the words "equals 5." The relationship among the blocks enables them to gain this insight. In contrast, the piecemeal act of counting single cubes often camouflages the meaning of the word *equal*.

Number Symbols and Signs: Level III

The teacher should introduce number symbols (numerals) after number concepts have been explored. The teacher fits the number guide into a slot at the top of the counting board (see Figure 11.1). When attached, it shows where each numeral comes in the sequence from 1 to 10. Each numeral appears above the groove that holds the number of cubes or the number block for which it stands (e.g., the 10 on the number guide appears above the groove that can hold 10 cubes or a 10-block). Between the number guide and the grooves are empty spaces for number markers bearing the numerals 1-10. On each number marker a line beneath the numeral indicates the correct orientation of that numeral (e.g., 6 indicates the marker is for 6, not 9). Having children match the numerals on the markers to the numerals on the number guide is an excellent check of the children's ability to perceive symbols and to position them correctly. This step prevents errors that might arise when they try to read or write numerals.

The teacher then lets the numerals "call back" the number blocks, an activity the children enjoy. The teacher says, "Pick a number. Where does it go?" The child selects a numeral such as 3, matches it to the 3 on the number guide, then finds the 3-block and fits it into the 3-groove.

Building Basic Concepts

Children who don't know math facts (e.g., 4+4 = 8) will have had difficulty learning them by rote and will resort to counting them out over and over again because they have become rigidly attached to the "counting song." Such children profit from remedial work with basic math concepts. When they realize they can learn basic concepts by playing exciting games with the counting board, they respond with improved concentration. One such game follows.

The Snake Game

The object of the Snake Game is to build the longest snake of number blocks. The teacher fills a counting board with number blocks and divides students into two teams of six to eight players. The teams take turns selecting a number marker from the face-down number markers that are scattered on the table. Each number marker indicates which number block the team may add to their snake of blocks. Children give up counting individual units on the number blocks when they realize it is quicker to name and select the number block below the number on the guide that matches the marker drawn. The winning team will have the longer snake of blocks.

The Snake Game prepares students to see and remember number combinations that they will discover when they later fill a number box with blocks (discussed later in this chapter). It is more impressive to note that working with counting board materials will enable children to move easily from addition to subtraction. But first they must be able to name the numbers and understand the concepts for which the number symbols stand.

Developing a Number Sense: Pattern Boards

Each *pattern board* provides between 1 and 10 empty blanks in a set pattern into which cubes may be inserted (see Figure 11.2). Children show that they recognize a number pattern by building it with cubes. They then check the pattern they have built by placing the cubes into the blanks of the correct pattern board. To learn the name of each number pattern, they count the cubes.

Even and Odd Numbers

The characteristics of evenness and oddness cannot be taught with number blocks, but they can be taught with cubes and pattern boards. Even numbers are formed by pairs of cube partners so that the pattern ends evenly. Children see that even numbers are made with 1+1, 2+2, 3+3, 4+4, and 5+5 (math facts that are taught later). When one cube is added to an even number pattern, it becomes odd. Children learn that "one cube all alone" distinguishes an odd number and that odd numbers are named 1, 3, 5, 7, and 9. A stick or pencil can be used to split an odd number pattern down the center to yield two consecutive numbers that form it.

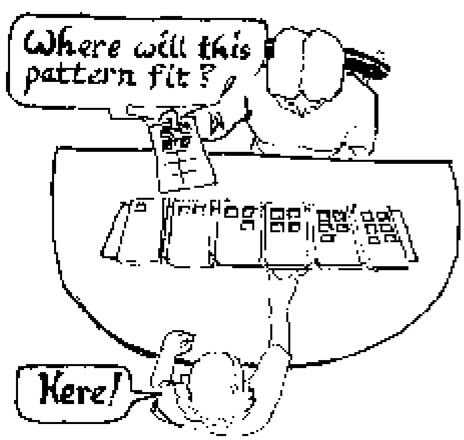

Figure 11.2. Pattern boards: matching cube patterns.

These odd number patterns are 1+0, 2+1, 3+2, 4+3, and 5+4. When building the sequence from 1 to 10, the odd number patterns are positioned between the even patterns. These facts are studied later under the name Doubles and Neighbors (discussed later in this chapter). The structure of these patterns makes the relationships visible and easy to recall later as addition facts.

Introduction to Word Problems

Acting out word problems with multisensory materials will prepare children to analyze complex word problems later. It is important for children to learn how to demonstrate word problems of their own creation. Pattern boards and cubes are excellent materials with which to begin. Each child in the group selects a different pattern board. For example, Laura selected the pattern board for the number 6, filled it with cubes, and then acted out her problem, saying, "My dog had 6 puppies. I gave 4 away. How many are left?" Even though she had subtracted 4 cubes, the children could still see the original number she started with because she chose a pattern board with 6 blanks. The pattern board allowed Laura's classmates to see at the same time the 4 subtracted cubes, the remaining 2 cubes, and their relationship to the original total of 6 cubes. Teachers know that some children, when taking 4 loose cubes away from 6 loose cubes, cannot visualize the original amount and wonder just how much they began with. They lose any sense of the relationship of 4 to 6 and of the remaining 2 to the 6 they began with. Well-planned lessons in which children act out word problems with multisensory materials such as pattern boards will help build strong foundations for later learning.

Addition and Subtraction with the 10-Box
Discovering Combinations that Make 10

As a first step, children fill the 10-box with pairs of number blocks as though they were working on a puzzle. The *10-box*, a square measuring 10 units by 10 units, has a frame around the edge that tells children when the total units of the block-partners is 10 (see Figure 11.3). Kindergartners often know which blocks go together in the 10-box before they can name them or designate them with number symbols. Each time they put a number block in the 10-box, they measure with their eyes to judge the size of the block that will fit with it. Thus they systematically study the relationship between each separate number block and the total, which is 10 units long. When they make a mistake, they can see and feel how the block does not fit. The teacher does not need to say, "Wrong!" The emphasis is on experimenting. When the children find pairs of blocks that fit, they feel a satisfaction that is unforgettable. This success with filling the 10-box helps them remember the block combinations that total 10 units. Once the children have learned the names of the blocks, they will find it easy to name the block partners that make 10 (see Figure 11.3).

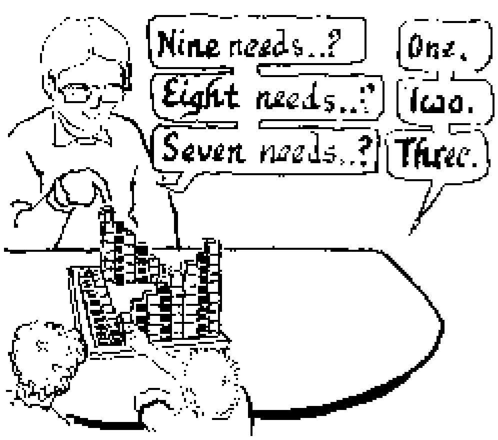

Figure 11.3. Filing the 10-box with number blocks.

The combination in the middle of the 10-box is a double, a 5-block and a 5-block. A double, children will discover, can be found in the center of every even number. In addition, students will understand the *zero fact* even before it is recorded; they can put the 10-block in the 10-box and say, "10 needs no other block to make 10." The children can show they understand the concept of equality when they place two number blocks next to the 10-block and say, for example, "8 and 2 are just as big as 10." This concept will be recorded later with the equal sign.

Building the Stair from 1 to 10 in the 10-Box

The teacher scatters number blocks 1-10 next to the 10-box and begins to build the stair in the lower left corner with the 1-, 2-, and 3-blocks. The teacher asks the children to complete the stair. When they put in a block that is too long (e.g., placing the 5-block after the 3-block), they realize that the structure is wrong and look for the correct block (e.g., the 4-block). The choices they make are guided by how the stair looks. The teacher can also challenge the children by removing one step of the completed stair while they are not looking and asking them to name the missing step.

Recording the Story of 10: Writing Equations

The goal is for students to record from memory the "Story of 10," the pairs of numbers that equal 10, but first they must learn how to record an equation after they fill a column in the 10-box. The teacher places the empty 10-box and two sets of number blocks before the children and sets to one side the number markers 1-10 and the plus and equal signs. The children put two number blocks (**addends**) into the 10-box and name them, for example, "6 and 4." The teacher records this by arranging the symbols 6+4 next to the 10-box. The students continue by saying, "equal 10," and the teacher records this by putting the appropriate symbols last, yielding 6+4=10. When the children later write equations, they understand the process from having recorded the combinations of blocks as they placed them in the box. Using the 10-box while recording equations also enables students to understand an equation when they read it.

For many children, the **missing addend** form of an equation (e.g., 6+___=10) is difficult to deal with as they do not understand what it means. When the children put a 10-block in the box and place a 6-block next to it, however, they find it natural to ask, "6 needs what to make 10?" They then record what they have just said with the number markers and symbols: 6+___=10. Students discover that the missing symbol is the numeral that stands for the missing block, which they know is the 4-block. Teachers often find that using the 10-box is the first time they have had an easy way of teaching equations with missing addends.

Subtraction and Its Relation to Addition

An addition example contains two or more addends, and students are asked for the sum or total; subtraction can easily be shown to have the opposite character. The children place a 6-block and a 4-block into the first column of the 10-box and note that they measure 10. The teacher states the addition fact 6+4 = 10. The teacher's next step is to explain how this addition fact will yield two subtraction facts. To demonstrate the first fact, the children realize that they begin with the total, 10, from which they subtract the 4-block; the 6-block remains. This demonstration gives meaning to the words, "10 take away 4 leaves 6," which students record as 10-4=6. The teacher then asks someone to demonstrate the other subtraction fact, 10-6=4, and to record it below the first. The structure of the materials makes it obvious that the smaller number is always subtracted from the larger number.

Combinations that Make 10

Playing the Hiding Game (described next) gives children more than practice in remembering the combinations that make 10. To discover which combination has been hidden, they must first figure out how the block combinations have been organized; they then find a strategy for identifying the missing combination. Afterward, it is interesting to ask students, "How did you do that?"

The Hiding Game

The teacher displays the 10-box filled with number block combinations in sequence from 1+9, 2+8, 3+7, and so forth up to 10+0. When the children's eyes are closed, the teacher removes one combination, such as 4+6, and says, "Open your eyes. What did I hide?" A child answers, "4+6," and says while pointing to the blocks in the lower stair, "I counted each step, 1, 2, 3, and found 4 was missing, so I knew 4 and 6 were missing."

Addition and Subtraction: Other Number Boxes

Children are delighted to find there are more number boxes smaller than the 10-box and enjoy fitting them inside one another to form a pyramid. On finding that the peak of the pyramid would be the 1-box, a precocious kindergartner called out, "I have the pièce de résistance!"

When asking the children to fill one of the smaller boxes, such as the 8-box, the teacher presents two sets of blocks as well as the box to be filled. Students will respond by rejecting the blocks that are too big (the 9- and 10-blocks) and finding the block combinations that exactly fit the 8-box. They realize that this task has the same features as filling the 10-box and adjust their performance accordingly. This adjustment comes about as a result of learning through insight, not by rote. Students enjoy playing the same addition and subtraction games in each box as they played in the 10-box. They later write the number story for each box and give each a title, such as the "Story of 8."

Different Roles a Number Can Play

Students will be excited when they become aware that a number plays a different role in each number box. Each number block's distinctive size and color enables children to observe how a number changes its role as it comprises the total of each different box. For example, consider the changes in the role of the 5-block. When filling the 5-box, a kindergarten child said, "5 is the control of this box!" When discovering the combinations while filling the 6-box, another child ran to his teacher and said, "Look! 5 is the 9 of this box." He had noticed that the 5-block plays the same role in the 6-box as the 9-block does in the 10-box: Both are the next-to-biggest block and thus join with a 1-block to fill a column in the box.

In addition, when putting together the combinations to make 9, an odd number, children find that there are two sets of consecutive numbers that make 9; they see that a 5-block joins a 4-block to make 9, and next to these blocks, a 4-block joins a 5-block to make 9. And finally, they discover that doubles are found only in the even numbers. Thus, when filling the 8 box, they find a pair of 4-blocks in the center.

Solving Word Problems

It is easy for children to understand when to solve a word problem by addition: There are two or more addends, and the total is being asked for. Children have greater difficulty identifying problems that are to be solved by subtraction. When a total is stated and a smaller amount is lost, it is relatively easy for children to decide to solve the problem by subtraction. Children, however, are more confused when trying to solve problems that say nothing about an amount being lost or taken away. In this type of problem a total and one amount are given, and children must figure out the other amount (e.g., The book costs $10. Tobias has $3. How much does he need to earn?). The children have to learn that the way to solve this type of word problem, a missing addend problem, is to subtract. Teachers usually start with problems in which the numbers are small, as in the previous example. Unfortunately, the use of small numbers makes it difficult for children to understand why they must subtract because they can so easily solve this kind of problem in their heads: "3 needs what to make 10? 7." This type of problem is especially difficult for children to solve later when the problems deal with big numbers (e.g., The television set costs $225. Tobias has $37. How much does he need to earn?) Children cannot solve 37+ ___=225 in their heads. The teacher must show them how to think about this type of problem by setting up a model so that when the numbers are big, it will make sense to subtract. For example, the teacher can put 10 cubes in a *number track* (a grooved track with numerals on one side that holds cubes or number blocks in sections of 10 units, or *decades*; see Figure 11.4) and say, "The book costs $10. Tobias doesn't need to earn all $10; he already has $3." The teacher covers 3 cubes with a scarf and says, "I covered 3 cubes to show that Tobias doesn't need to earn those $3; he already has them. I've taken 3 away from 10." The teacher writes 10-3=___ and says, "Tobias needs to earn the rest." The children conclude in a full sentence, "Tobias needs to earn $7." The children now have a model of how to solve missing addend problems. Later, they will be able to substitute big numbers in this model. For example, "Ted wants $225. He has $37. He doesn't need to earn it, so I'll take it away! He needs to earn the rest!" They will understand that missing addend problems are solved by subtraction and will set them up correctly.

Teen Numbers

It is important for the teacher to show mathematical concepts such as teen numbers in more than one way. The teen numbers are the numbers above 10 through 19, composed of a 10-block and a number of ones (the rest of the number). They will be shown first in the *20-tray* (a square tray similar to a 10-box, which measures 20 units by 20 units), then in the number track (see Figure 11.4), and also in the *dual board* (see Figure 11.5).

Figure 11.4. Building 14 in the number track.

Building the Stair from 1 to 20

In the 20-tray, children begin by building the familiar stair from 1 to 10. On reaching 10, they discover they must piece together the next numbers using a 10-block and another number block (e.g., 1 ten and a 1-block to make 11, 1 ten and a 2-block to make 12, 1 ten and a 3-block to make 13). Students will be surprised when they see that the stair from 1 to 10 repeats itself in the teen numbers on a base of 10-blocks. They name each number in the 20-tray by naming the steps from 1 to 20. To make certain that the students understand the structure of teen numbers, the teacher says, "Close your eyes," and hides the blocks composing a teen number such as 14. When the children look at the stair of teen numbers, they see which step is missing and cry, "14!" As the teacher replaces the number blocks in the missing step, the students name them: "10 and 4." After this activity they will think of the number 14 as the teacher has built it, with 1 ten and 4 ones (a 4-block). Visualizing the structure and naming the blocks enables students from the start to write the digits in the correct order as 14, not as the number is pronounced, "Four-teen."

Measuring Teen Numbers in a Number Track

When the number blocks for a teen number are placed in the number track, the numerals on the side of the track record the total. Children need to be familiar with the structure of a teen number in the number track.

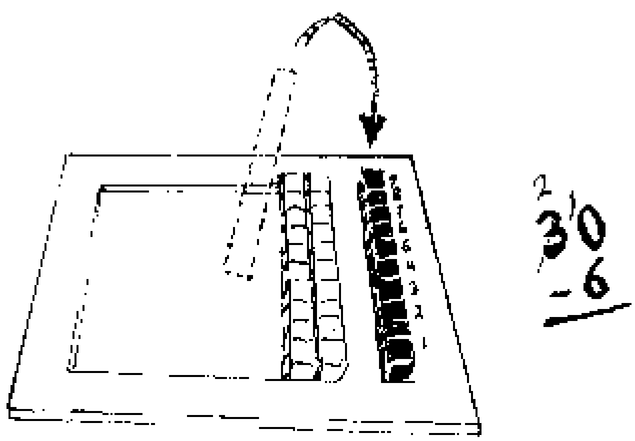

Figure 11.5. A dual board. The example 30-6 is worked out by carrying back 1 ten to the ones compartment, exchanging it for ten ones, and subtracting six.

Operating from this base, they will easily understand the next step, that of adding to 9, in which they first add a number block, such as a 3-block, to a 10-block and then to a 9-block. They can see that the 10-block and the 3-block reach 13 but that the 9-block and the 3-block only reach 12.

Building Teen Numbers in the Dual Board

The dual board has two compartments, a large "tens" compartment the same size as a 10-box, and a small "ones" compartment that holds a column of ten 1-blocks. Before using the dual board for purposes such as subtracting from 2-place numbers (see Figure 11.5), it is important for children to construct the teen numbers in the dual board.

When children first look at a written number such as 12, they see two numerals that seem to stand for numbers of about the same size. By building a number in the dual board, they gain insight into the important role that the *place*, or the position, of each numeral plays. The teacher builds a number, such as 12, in the dual board by placing 1 ten in the tens compartment and a 2-block in the ones compartment. To record this number, the number marker 1 is put below the tens compartment and the number marker 2 is put below the ones compartment. This shows the meaning of each symbol in the written number 12. To make the term *place value* clear, the teacher can change the places of the number markers 2 and 1 and have children show the new value of each symbol, that is, 2 tens and a 1-block.

Addition Facts with Answers in the Teens

There are 36 addition facts with answers in the teens (e.g., 9+2=11 to 9+9=18). These facts can be categorized into different groups of structurally related facts: Adding 9, Adding to 9, Combinations that Make 11 and 12, and Doubles and Neighbors. Children master them by understanding the general rules for each group.

Adding 9

There are eight facts in the Adding 9 group (2+9, +9, 4+9, 5+9, 6+9, 7+9, 8+9, 9+9). Children can master these eight facts at once by learning only one general rule. First they add a 10-block to any number, such as 3, in the number track and realize that the digit in the ones place of the total, 13, is the same (e.g., 3+10 = 13). Students then add a 9-block to a 3-block and discover that they only reach 12, a number that is 1 less than if they had added a 10-block to the number they started with (e.g., 3+10=13, but 3+9=12). Students demonstrate that they understand these facts by adding a 9-block to each number block from 2 to 9.

Adding to 9

Seven addition facts result from adding a number to 9. To learn these Adding to 9 facts, which are the reverse of the Adding 9 facts, children add blocks to a 9-block. This time, the teacher places a 9-block in the number track, holds a 3-block above the track, and asks, "What do you think 9 1 3 equals?" The children have time to answer before the teacher places the 3-block after the 9-block in the number track. When the blocks are joined end to end, the children can see that the 3-block had to move down one unit to fill the gap between 9 and 10; thus, adding 3 to 9 reaches the number 12 (i.e., 10+3=13, but 9+3=12). They figure out all of the Adding to 9 facts adding the numbers 2 through 8 to 9.

Combinations that Make 11 and 12

The children begin by building a stair from 1 to 10 in the 10-box. They then add the top stair of blocks to this stair so that each column in the 10-box is filled. Now they move the whole top stair up one step. This results in combinations that make 11. The children know the combinations that make 10, so when making combinations that total 11, they realize that one of the component parts will be one unit bigger. For example, 5+5=10, but 5 needs 6 to make 11; 7+3=10, but 7 needs 4 to make 11.

To build combinations that make 12, students again move the blocks of the top stair up one step. This time 2 units must be added to one of the component parts that totaled 10; for example, 5+5=10, but 5 needs 7 to make 12. The teacher can now use the clock on the wall as an unforgettable way to show the combinations making 12. The numbers that stand across from each other on the clock face add up to 12. For example, 9 is across from 3, and 9+3=12; 8 is across from 4, and 8+4=12; and 7 is across from 5, and 7+5=12. Once the children have learned Adding 9, Adding to 9, and Combinations that Make 11 and 12, they have studied two thirds of the addition facts with sums that are teen numbers.

Doubles and Neighbors in the Teens

The next group of structurally related addition facts is called Doubles and Neighbors in the Teens and consists of six new facts. Children usually find the doubles (6+6=12; 7+7=14; 8+8=16; 9+9=18; 10+10=20) easy to learn. To illustrate the doubles, the teacher places pairs of like blocks side by side and explains that the sum of each pair is an even number. For practice in learning doubles, children should play a game such as Stop and Go (described later in this chapter). To illustrate the facts that are neighbors of the doubles (5+6=11, 6+7=13, 7+8=15, 8+9=17, 9+10=19), the teacher places the pair of 6-blocks (12) next to the pair of 7-blocks (14) and leaves a space between the pairs. The teacher points to this empty space. To make the neighbor between 12 and 14, a child takes a 6-block from the pair that totals 12 and a 7-block from the pair that totals 14. This produces the number 13, built from the consecutive numbers 6 and 7. The teacher demonstrates how each odd teen number can be formed from two consecutive numbers. Once students have mastered Doubles and Neighbors in the Teens, there are only four final addition facts with answers in the teens to be learned: 8+5 and 8+6 and the reverse facts, 5+8 and 6+8.

Addition and Subtraction Facts in the Teens

Teachers often must work with children who find math especially difficult to comprehend. The teacher must first discover which concepts these students do not understand. Looking at students' final answers to math problems does not indicate their level of understanding. For example, if the teacher sees that a student wrote 15-9=6, then the teacher still does not know how the student figured out the answer to this example. If the teacher, however, sees that the student made 15 dashes, crossed off 9 of them, and counted the remaining dashes, then the teacher has gained valuable information. Students who have difficulty with math concepts need to work with materials structured so as to give them the foundation they never had to understand computation with answers that are teen numbers.

First, they must build the stair of teen numbers in the 20-tray using 10-blocks and unit blocks to construct the numbers 11-20. Second, they must build a teen number such as 15 in the number track using a 10-block and a 5-block. Then by substituting the 9-block and the 1-block for the 10-block, they should conclude that the result of taking 9 from a teen number such as 15 is easy to figure out. When the 9-block is removed, the 1-block remains in the number track with the 5-block to yield the answer, 6. Children find it easy to demonstrate similar examples with the same structure: 17-9=8 or 16-9=7. This kind of insight gives children confidence in their ability to answer difficult math problems without the endless counting of dots or dashes. In particular, children with learning disabilities profit most from such experiences. They need to realize that they can think and that they can do arithmetic.

Games with Teen Numbers

It is important for all children to learn the concepts about teen numbers and to practice them by playing games that they find challenging.

The Hard Snake Game

The Hard Snake Game may be played by two teams or two children. On each of 10 cards, the teacher writes a teen number and turns it face down. One member of Team A then selects a card and announces the number written on it. The player must form this total by adding two blocks, one of which must be a 9-block. Thus, if the card said 15, then the player must build 15 using a 9-block and a 6-block. If the player forms the correct total, the blocks are then added to Team A's snake of blocks. The winner is the team with the longest snake of blocks. This game reinforces the students' learning of these difficult teen number facts.

The Probe Game

To prepare for the Probe Game, the teacher displays the number track from 1 to 20 and covers the numbers 1-10 with a cardboard box. The teacher has the children close their eyes and then hides a number block, perhaps the 5-block, under the cardboard box. The teacher lets the students open their eyes and asks a child to discover the size of the hidden block by using the 9-block as a probe. As he or she pushes the 9-block down the track and under the box, the children wait expectantly until it stops and shows that the sum is 14. The child must reason, "If this were the 10-block, then the hidden block would be the 4-block, but it is only the 9-block, so the hidden block has to be one bigger than 4: it must be the 5-block." At this point the teacher verifies the answer by lifting the cardboard box to reveal that the hidden block is indeed the 5-block.

Place Value or Positional Notation

Giving up dealing with ones and grouping them into tens as the new unit of measure is one of the most ingenious ideas of mankind. Here is an experiment that will show the children how the use of place value, or *positional notation*, helps us deal with big numbers. The teacher throws only a few cubes on the table and explains that it is easy to say how many are in a small group of cubes. Next, the teacher throws more than 50 cubes onto the table and writes down a few of the children's estimates of the amount. To determine the exact number, the teacher arranges the cubes in rows of 10 in the tens compartment of the dual board and places any leftover cubes in the ones compartment. Students can see at a glance that the amount of cubes is 5 tens plus the 4 leftover cubes in the ones compartment (i.e., 54). The teacher can explain that grouping objects in tens and ones is one advantage of the structure of our number system.

Recording Teen Numbers and Understanding the Numeral 0

When students built the teen numbers, they discovered that the composition of each number corresponded exactly with its numerals: 1 ten and 1 one were recorded as *11*, 1 ten and 2 as *12*, and so forth up to 1 ten and 9 as *19*. The teacher asks the children to put two 10-blocks in the dual board and record it by placing the number marker for 2 below the tens compartment. The teacher then places the marker for 0 below the empty ones compartment and explains why 0 is called a *place holder*: "If we write the numeral 2 and let it stand alone, it would mean 2 ones. So we write 0 in the ones place to tell us that this time the digit 2 stands for 2 tens and that there isn't anything in the ones place. Moreover, in 20, the 0 is also holding a place for another numeral. If we replace 0 with 1, we must also put 1 cube in the ones compartment." The teacher explains that the numeral is now 21, records the amount in the dual board, and says, "2 tens, 1 one."

Building 2-Place Numbers in the Dual Board and the Number Track

Children are now ready to discover the meaning behind the structure of the 2-place, or 2-digit, numerals between 10 and 100. They first build a number such as 25 with 2 tens and 5 ones, then they reverse the digits and build 52, using 5 tens and 2 ones. They are now ready to play an interesting game.

Build Your Guess

The game Build Your Guess gives children practice building 2-place numbers in the dual board and then measuring them in the long number track to note where they come in the sequence of 1 to 100. To begin, the teacher writes a 2-place number on a piece of paper and hides it. The student who comes closest to guessing the teacher's number is the winner. Each child should guess a different number and must then build his or her guess in the dual board. For example, Erin decides to guess 39, so she puts 3 tens in the tens compartment and a 9-block in the ones compartment and records it with number markers for 3 and 9. She then measures her blocks in the number track; they reach 39. She places her number markers, 39, next to the track to show her guess. Then the teacher shows the hidden number, 36. Erin finds that her guess, 39, is 3 units bigger than 36, whereas Toshi's guess, 32, is 4 units smaller than 36. The others were even further away. Erin concludes she is the winner.

In this game and the other Structural Arithmetic activities, the children have been expressing their ideas by building numbers with blocks and recording them with numerals. This solid foundation in understanding 2-digit numbers will make regrouping in addition and subtraction (i.e., carrying and borrowing) easier for them to understand.

Regrouping in Addition

The procedure called **regrouping** is known to previous generations as *carrying* and *borrowing*. The teacher explains that in our system of positional notation, when the amount in one column exceeds 9, it can only be expressed by a 2-digit numeral that has a tens digit and a ones digit; therefore, the amount must be regrouped. For example, when dealing with an amount in the ones column such as 12 ones, 10 ones must be exchanged for or regrouped as 1 ten and carried to the tens column. That leaves 2 in the ones place. The numeral that records 1 ten and 2 ones is 12.

When children work with the blocks in the dual board, these procedures will have meaning from the very start.

Students learn addition with regrouping step by step using the dual board. The first step in adding two numbers, such as 39 and 1, is to build the 2-place number in the dual board. The children place 3 tens in the tens compartment and 9 ones in the ones compartment and say the name of the number, "Thirty-nine." The teacher adds 1 cube to the 9 cubes that are already in the ones compartment and explains, "We cannot record the 10 cubes in the ones compartment with a single numeral; we must regroup!" Students exchange the 10 cubes for 1 ten, carry it to the tens compartment, and add it to the 3 tens already there, resulting in a total of 4 tens.

Once the children have understood regrouping, they record each step on paper; writing on paper lined in half-inch squares is a great help. The children write 39, then they write + 1 below the 9 in the ones column, draw a horizontal line beneath it, and say, "9 plus 1 equals 10. I must regroup." They reenact the carrying of 1 ten to the tens compartment and record this move by writing a small figure 1 above the 3 in the tens column of their example. They write *0* in the ones place below the horizontal line (because there are no ones left). Next they add the digits in the tens column and say, "3 plus 1 equals 4." They write 4 in the tens place in their answer. The sum is written as *40,* which accurately records the 4 tens in the dual board.

Next, the students add a bigger number, perhaps 3, to 39. When they add three cubes to the 9 that are already in the ones compartment, they see that they have twelve cubes and must regroup. They exchange ten cubes for 1 ten and carry it to the tens compartment to join the 3 tens. That move leaves 2 cubes in the ones compartment. The students record each step on paper and get the answer, 42, which they see accurately records the 4 tens and 2 ones in the dual board.

Money and Regrouping

To make clear the value of different coins, the teacher tapes a dime to each of several 10-blocks, a nickel to each of several 5-blocks, and a penny to each of about 20 cubes. Children are now able to see the relationship among the values of the different coins. The nickels are taped to blocks that are half the size of the blocks to which the dimes are taped. Students can see that nickels, which are larger in size than dimes, are worth less than dimes and that it takes 10 pennies (also larger in size than dimes) to equal a dime, or 10 cents. The children will enjoy using the money on blocks in a store game. They can also use the money on blocks in other games.

Score 1 If You Carry

This game is played with the dual board, money on blocks, and a die. The teams take turns tossing the die and placing pennies in the ones compartment of the dual board. For example, Team A tosses the die, gets 3, and places 3 pennies in the ones compartment. Team B tosses a 4 and adds 4 pennies, bringing the total to 7. Team A tosses the die and gets 5, which, when added to 7, yields 12 pennies. Team A must regroup; it exchanges 10 pennies for 1 dime and carries it to the tens compartment. The team puts the 2 remaining cents in the ones column. Team A "scores 1" because it got the chance to regroup, or "carry 1." The game continues. The game could end when one team's score is 5.

Regrouping in Subtraction

The process of regrouping in subtraction has also been known as borrowing. The children again work in the dual board. They write 30 on their papers and build it with 3 tens. The teacher dictates, "Subtract 6." The children write - 6 below the 0 in *30* and draw a horizontal line below the subtraction example. The teacher explains that the students cannot take 6 cubes away from the ones compartment of the dual board because there are no cubes there and that they must regroup. They carry back 1 ten from the tens compartment and exchange it for 10 ones. On their papers they cross out the 3 and write a small 2 above it to indicate that there are only 2 tens left. The students write a small figure 1 above and to the left of the *0* in the ones place to indicate they now have 10 cubes in the ones column (see Figure 11.5). Now they can subtract 6 cubes, which leaves 4 cubes. The final amount is 2 tens and 4 ones, which matches the written answer, *24*.

Regrouping with the Dual Board

After working with the blocks in the dual board, the children need to practice regrouping in a more exciting situation, such as a game.

Your Answer Is Your Score

During each turn in the game Your Answer Is Your Score, one player selects two face-down number markers and uses them to build a two-digit number with number blocks in the dual board. He or she must then subtract from this number an amount that is large enough to require regrouping. The player's score is the number that remains after this subtraction operation; the player with the highest score wins. For example, Robert, who has selected the markers 6 and 3, understands that the goal is to get a big score, so he explains, "I don't want to build 36; I'll build 63, of course!" The teacher asks, "How much will you subtract from 63 so that you must regroup?" Robert reasons, "I have only 3 cubes in the ones compartment, so I think I'll subtract 4 from 63." He exchanges 1 ten for 10 ones and puts them into the ones compartment; he now has a total of 13 single cubes. He subtracts 4 cubes from the 13, and this leaves 9 cubes. Robert sees that there are now 5 tens and 9 ones in the dual board and exclaims, "My score is 59!" He writes 59 on the chalkboard. The rest of the players will try to beat his score.

Multiplication

To most people, *multiplication* still means the memorization of 100 facts such as 7x6=42, which are repeated again and again until they have been learned by heart. Actually multiplication and division are fascinating operations that, when studied with the Structural Arithmetic materials, allow children to discover new relationships among numbers. They come to realize that they are viewing the same facts from opposite sides when they discover that the relationship between multiplication and division is one of doing and undoing, just as it is between addition and subtraction. Students will not only grasp this interrelationship but also understand, through their experiments with the materials, just how division is the opposite of multiplication.

The operation of multiplication can, of course, be taught by the addition of equal addends. For example, the answer to 5x2 can be figured out by addition: 2+2=4, 4+2=6, 6+2=8, and 8+2=10. In Structural Arithmetic, however, children produce an amount a given number of times, and by measuring it in the number track, they discover how this total is expressed in our base-10 system, that is, with tens and ones.

Discovery of What a Multiplication Table Is

The children will discover that multiplication is a new way of expressing number relationships. The essence of multiplication is that an amount is taken not once but several times. Therefore, to find the answer to 5x2, the children take a 2-block five times. The number 5 is not represented by a block but is the **operator** and plays a different role from the 2; it indicates how many times 2 is to be produced.

Multiplication tables are often recited by saying, for example, "5 times 1 equals 5, 5 times 2 equals 10, 5 times 3 equals 15," and so forth. To represent these facts with materials would mean changing the size of the number blocks used each time: five 1-blocks, then five 2-blocks, then five 3-blocks, and so forth. It makes more sense to demonstrate the multiplication table using the same size number block each time: one 5-block, then two 5-blocks, then three 5-blocks, and so forth (see Figure 13.6). By following this procedure children will be able to discover the products of any multiplication table by themselves. They can take a block a certain number of times and measure the total in the number track. They can find, for example, the way that three 5s are expressed in our base-10 number system: 1 ten and 5 ones, or 15 (see Figure 11.6). Students discover not only the interrelationship among the facts within one multiplication table (e.g., 1x5, 2x5, 3x5, and so forth up to 10x5) but also the interrelationship among the facts of different tables, such as the multiples that two or three tables have in common.

One approach used to teach children to represent a multiplication fact is to have them draw circles and stars. To demonstrate 3x4, they make three circles and put four stars in each circle. This arrangement shows the role of the operator, or **multiplier**, 3, and the size, 4, but does not show how the total is expressed in the denominations of our number system, tens and ones. There is no visible connection between the fact 3x4 and the product 12, which can only be found by counting. The same is true when children create arrays, such as 4 rows of 5 dots each.

Sequence of Teaching the Multiplication Tables

Children begin by studying the 10-table because our number system is built on base 10. When the **multiplicand** is 10, each product can be expressed immediately. The product of 3x10, 30, does not have to be calculated but can be shown by writing 3 in the tens place of the numeral and adding a *0* as a placeholder; the multiple is already expressed in tens and ones. The need for multiplication tables arises because for any multiplicand other than 10, the products must be expressed in tens and ones.

The multiplication facts in the 1-table can be shown by producing the 1-block a certain number of times (5x1) or each number block one time (e.g., to show 1x5, a student produces a 5-block); the answer is the quantity the number block itself represents. The 2-table is studied next because its characteristic feature is that a given number of 2s is the same as the double of that given number (e.g., 6x2=12 and the double of 6 is 12). Because the children know the doubles, they easily master the 2-table . The 5-table is studied next because of the special relationship of 5 to

10 (5 is half of 10), followed by the 9-table because of the closeness of 9 to 10 (9 is 1 less than 10).

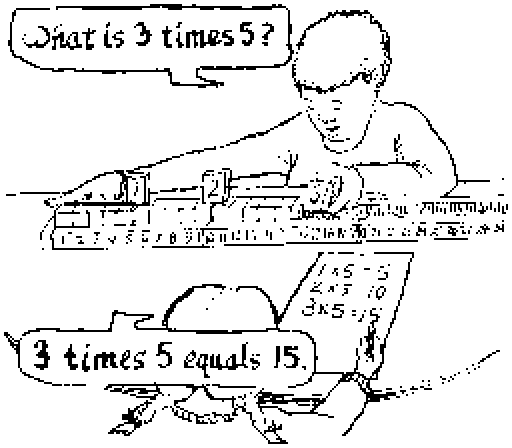

Figure 11.6. Discovering what 3 times 5 equals.

To teach the 9-table, the teacher puts one 10-block in the number track and writes 1x10 = 10. The 10-block is then replaced by a 9-block and a 1-block. The teacher then subtracts the 1-block and writes 1x9 = 10-1; beneath that he or she writes, 1x9=9. To show the second multiple of 9, the teacher places two 10-blocks in the track and writes 2x10=20. Next, the teacher replaces the 10-blocks with two sets of 9-blocks and 1-blocks. Both 1-blocks are subtracted, and the two 9-blocks are shoved together; they reach 18. The teacher writes 2x9 = 20-2. Below that he or she writes 2x9 =___; the children copy the fact and write 18 in the blank. By following this procedure the teacher and the children continue to demonstrate and record the 9-table and finish with *10x9=100-10; 10x9=90.*

Multiplication Games

Stop and Go

Stop and Go is one of several exciting games that children can play to reinforce their understanding of a multiplication table. To play Stop and Go with the 5-table, the teacher fills the number track with 10 of the 5-blocks. He or she assigns a red cube to the Red Team and a blue cube to the Blue Team and places them at the beginning of the track. The stop-and-go cube has two red "Stop" sides and four green "Go" sides. The Red Team drops the stop-and-go cube and gets Go. The Red Team places its cube on the end of the first 5-block above the number 5 on the track, and one of the Red Team's players says, "1 times 5 equals 5." The Blue Team also gets Go, so it places its blue cube piggyback on top of the red cube. Here are the two possible moves for the Red Team: If Red gets Go, it must carry the Blue Team's cube, move both cubes up to the end of the number 10 on the number track, and say, "2 times 5 equals 10." If the Red Team gets Stop, however, both teams' cubes remain on 5. If the Blue Team gets Go at the next turn, it will move off of the red cube. In this way, they race each other up the track landing on each multiple of 5 until one team reach the 10th multiple, 50.

When studying the multiples of the 5-table in the number track, as with Stop and Go, the children will discover that each decade in the number track holds 2 fives. Therefore, when the multiplier is even, the product ends in 0 because it equals a number of 10s; when the multiplier is an odd number, the product has 5 in the ones place because an odd number of 5s reaches the middle of a decade: 1x5=5, 3x5=15, 5x5=25, 7x5=35, 9x5=45.

Capture Peaks

Capture Peaks is another favorite game. First, the children choose a multiplication table and place 10 of the appropriate blocks in the number track. For example, the children decide on the 4-table and place ten 4-blocks successively in the number track. Then they dictate each fact for the teacher to write on the chalkboard. On each of 10 dominoes a fact is printed in this form: 1x4=___. The Red and Blue Teams each get 10 cubes of their respective color, which thus limits each team to 10 turns. A member of the Red Team turns up one of the face-down dominoes and reads it aloud: "3 times 4 equals 12." Then the team places its red cube on 12 in the number track. The domino is then returned to the pool so that each multiple in the number track can have more than one cube on it. After several more turns a member of the Blue Team might get 3x4=___ and say, "3 times 4 equals 12! I'm on top of the Red Team!" At the end of the game, the color cube on top of each peak or multiple determines which team claims the entire tower of cubes beneath it. The team with the most cubes wins.

The Screen Game

The Screen Game, which focuses on the use of words in math, builds a foundation that will enable students to understand similar wording when they encounter it later in algebra problems. Children who have had difficulty understanding the meaning of words sometimes mistake the words "3 times as big" to mean "3 times bigger than" and thus produce an amount "3 times as big" as the original and then add this tripled amount to the original amount. The result is an amount that is 4 times as big as the original. For children who make similar mistakes, the Screen Game is an important activity.

To begin, the teacher suggests that the children watch the Screen Game carefully so that they can invent similar problems of their own. Then the teacher writes a few math expressions on the chalkboard, such as *times as big as, times as long as, times as much as, times as heavy as, and times as old as.* The children then invent their own problems in which they use one of the expressions.

The teacher gives the children a pile of number blocks and stands a screen on a table. He or she places a 6-block in front of the screen and says, "This represents a stick 6 feet long-perhaps 6 doll-feet! In back of the screen is a stick 3 times as long. Show me with the blocks how long the hidden stick is." If the children have understood the words "3 times as long," they will build a stick with 3 of the 6-blocks. The hidden stick is then revealed for comparison. The children write their equations on paper (3x6= *18*) and say, "The stick is 18 feet long." The teacher then asks several children to demonstrate a problem of their own.

For example, Chan invented this problem: "My dog weighs 20 pounds. I weigh 3 times as much. How much do I weigh?" The teacher had to help another student, Jim, to come up with the wording of his problem: "My father is 4 times as old as I am. I am 10 years old. How old is my father?"

After having students come up with their own problems, such as in the Screen Game, teachers often reflect that they have been too interested in leading every activity themselves. They realize that they must learn to stand back and encourage the children to develop and use their own powers of creation.

The Relationship Between Muliplication and Division

Students who have learned multiplication with the Structural Arithmetic materials have experienced for themselves how different the role is that each number in an equation plays.

In a multiplication example, such as 3x5=___, the multiplier, 3, and the multiplicand, 5, are given; the product is to be found. In one of the related division examples, the total, called the **dividend**, is given and has to be divided into a given number of parts. The question is "What is the size of each part?" (15÷3=___). This is the *partition aspect* of division. In contrast, when the total and the size of the part are given, the question is "How many times is the size, or part [5] contained in the dividend?"

$5\overline{)15}$

The answer, 3, is called the **quotient**. This is the *containing aspect* of division.

Students can take the idea that three 5s make 15, turn it around to answer the new question, "How many 5s are in 15?" and immediately answer, "3." By contrast, children who learn the 100 multiplication facts by rote acquire a habit of absent-minded mechanical activity that does not allow them to see the relationship between multiplication and division.

The Partition Aspect of Division

Most children experience the partition aspect of division at home whenever food or toys are divided among a few children. For example, a total, such as 15 cookies, is divided among 3 children; the answer to the question "How many cookies will each child receive?" (5) is the answer to the division problem 15÷3. The children can carry out many such problems and record them (e.g., 15÷3=5).

The Containing Aspect of Division

When considering the partition aspect of division, the total and the number of shares are stated, and the question is to find the size of the share. In contrast, when considering the containing aspect of division, the total and the size of the part are given. The question is to find the number of shares (e.g., "How many times is the part contained in the total?"). For example, the previous problem changed to demonstrate the containing aspect of division would be stated like this: "Mother has 15 cookies. She wants to give 5 cookies to each child. How many children can she give cookies to?"

The containing aspect of division is taught first because it makes the structure of the division algorithm so clear. Children find they can solve a division example as soon as they understand multiplication.

The **division radical** looks like this: $\overline{)\quad}$ and can be presented so as to make sense to students. The teacher places a small paper division radical on the number track over a multiple, such as 15. By doing this the teacher shows how the number 15 changes roles from being the product in a multiplication example to being the **dividend** in a division example. When the children see a division example, such as

$5\overline{)15}$

The teacher explains that this asks, "How many 5s are in 15?" To answer this question, students lay their pencils across 15 on the number track and find it takes 3 of the 5-blocks to reach 15, thus 3 tells them "how many times" the **divisor** (5) are contained in the dividend (15). The children then place the number marker for 3 on top of the number track above the number 15. This placement reminds them to write the quotient above the division radical when they record the answer to a division example (see Figure 11.7). By demonstrating examples such as this one, the children begin to understand the relationship between multiplication and division.

Under the Box

The game called Under the Box helps children remember which number in a division example is written "under the box" (under the division radical). When children hear words such as "24 divided by 4," they often write the numbers in the order in which they hear them, like this:

$24\overline{)4}$

The teacher hides several number blocks of the same size under a box, such as four 5-blocks, writes the total on a card, and says, "The blocks under the box make 20." The children write 20 "under the box" on their papers, like this:

$\overline{)20}$

The teacher continues, "The blocks under the box are 5-blocks. How many 5s are there?" The students ask themselves, "How many 5s are in 20?" and write

$5\overline{)20}$

They answer, "4," and lift the box, which reveals the four 5-blocks. They write 4 above the division radical:

$$\begin{array}{r} 4 \\ 5\overline{)20} \end{array}$$

Division with Remainder

When long division is presented with the Structural Arithmetic materials, children can see that each step of the process makes sense. The teacher dictates a division example that will have an answer with a remainder, such as "How many 5s are in 14?" (see Figure 11.7). The children place two 5-blocks in the number track plus 4 cubes to reach 14, and write

$$5\overline{)14}$$

On their papers. The children see that there are 2 fives in 14, so they write 2 above the division radical, but they know that 2x5 is only 10. The teacher explains that now the students should find the number of cubes left, so he or she takes the 2 fives out of the track. The children say, "Subtract 10 from 14," and they write -10 below *14* in their examples:

$$\begin{array}{r} 2 \\ 5\overline{)14} \\ -10 \end{array}$$

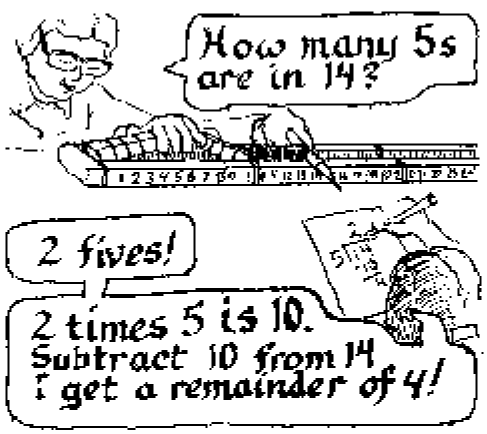

Figure 11.7. Division with remainder.

Because 10 has been subtracted from 14, 4 cubes remain in the number track. Thus, the expression "remainder of 4" makes sense to the students. They complete their work by writing the remainder in their answers:

$$\begin{array}{r} 2\,R\,4 \\ 5\overline{)14} \\ -10 \end{array}$$

Division Word Problems

Division (Containing Aspect) 15 children want to go on boat rides (total). One boat holds 5 children (size of part). Question: How many boats will they need? We want to find how many 5s are contained in 15, so we write

$5\overline{)15}$

They will need 3 boats. We can check this: 3x5=15.

Division (Partition Aspect) 15 children want to go on boat rides (total). There are only 3 boats. Question: If there are an equal number of children in each boat, how many children are in each boat? We want to know the size of the part or the number of children in each boat. [To show that the students are finding the size of the part this time, the teacher can use a different way of writing the example, such as 15÷3=5.] There will be 5 children in each boat.

Conclusion

This chapter shows how the multisensory Structural Arithmetic materials enable children to discover mathematical concepts and relationships that they could not have discovered through counting procedures and memorization alone. Children often respond appreciatively when a demonstration gives them insight into a new concept by exclaiming, "How neat!" Such enthusiasm is quickly responded to by the other children and gives the teacher special pleasure in working with these materials.

Today educators and parents are concerned about the use of computers by children. What effect will their use have on the ability of children to develop intellectual competence and to learn effectively. Jane Healy has made extensive studies of children working with computers at different ages. In her latest book, Failure to Connect, she writes: "It is no accident that formal schooling in most countries begins at the time when the brain is sufficiently organized to grasp abstract symbols. Prior to that time, young children are 'concrete learners': They need to 'mess around,' experiment, and create meaning with their own symbol systems. Preschoolers don't learn language and concepts from two-dimensional flash cards, but from multidimensional experience." Premature reliance on abstractions, on visual images and 'mouse' movements and 'clicks' without sufficient exposure to 3-dimensional experience is likely to diminish the density of meaning children associate with symbols. To the extent that computers may deprive young children of such experience, or cut dramatically into the time they spend interacting with the concrete world, the symbols may come to hold less tangible associations.

Piaget (1952) explained that children develop an understanding of mathematical concepts as a result of the actions they perform with objects, not as a result of the objects themselves. This makes clear why pictures in workbooks do not teach children. The Structural Arithmetic blocks invite children to handle them and perform actions at every step. By the time students reach the level of writing in workbooks, they are free to focus on how to write numbers and equations; they have already worked out the concepts with the multisensory materials and are ready to record them.

By using these structured materials, children, especially children with learning disabilities, learn to trust their intelligence; they come to feel pleased with themselves and proud of their ability to think. This feeling of self-confidence increases their self-esteem and often spreads to their work in other fields. Furthermore, they have developed a reliable understanding of number concepts from the very beginning. They will not have to discard these early formulations as deceptively simple but can carry them forward as the building blocks of algebra.

When we teach mathematical truths and facts by rote, we take away from children not only the joy of using their minds but also their sense of independence. They feel manipulated, and the result is unimaginative mechanical work. In some cases this causes students to be bored; in other cases, this leads students to turn off their minds, to withdraw, or to fail. What children crave is freedom to carry out their own experiments and draw their own conclusions. When they are permitted to do this to the degree that they can, their work will be fulfilling.

References

Healy, J. *Failure to connect* (1999). A Touchstone Book, Simon & Schuster Inc.
Piaget, J. (1952). *The child's concept of number*. London: Routledge & Kegan Paul.
Stern, C., & Stern, M. (1971). *Children discover arithmetic: An introduction to Structural Arithmetic*. New York: HarperCollins.
Stern, M. (1988). *Experimenting with numbers: A guide for preschool, kindergarten, and first grade teachers.* Cambridge, MA: Educators Publishing Service.
Stern, M., & Gould, T. (1988-1992). *Structural arithmetic workbooks 1-3 and teachers' guides.* Cambridge, MA: Educators Publishing Service.
Stern, M. (2002) Web site: http://www.SternMath.com